

Exercise 38

Find the derivative. Simplify where possible.

$$f(t) = \frac{1 + \sinh t}{1 - \sinh t}$$

Solution

Take the derivative using the quotient rule.

$$\begin{aligned} f'(t) &= \frac{d}{dt} \left(\frac{1 + \sinh t}{1 - \sinh t} \right) \\ &= \frac{\left[\frac{d}{dt}(1 + \sinh t) \right] (1 - \sinh t) - \left[\frac{d}{dt}(1 - \sinh t) \right] (1 + \sinh t)}{(1 - \sinh t)^2} \\ &= \frac{(\cosh t)(1 - \sinh t) - (-\cosh t)(1 + \sinh t)}{(1 - \sinh t)^2} \\ &= \frac{\cosh t(1 - \sinh t) + \cosh t(1 + \sinh t)}{(1 - \sinh t)^2} \\ &= \frac{\cosh t(1 - \sinh t + 1 + \sinh t)}{(1 - \sinh t)^2} \\ &= \frac{2 \cosh t}{(1 - \sinh t)^2} \end{aligned}$$